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14. ABSTRACT
Two students, Arrash Hassibi and Meeko Oishi, were supported during the period of this grant. Dr. Hassibi worked on several topics, all related to the topic of the AASERT award and its parent AFOSR grant Meeko Oishi worked on the topic of switching in nonminimum phase systems with applications to a VSTOL aircraft, as relates to hybrid systems.

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AASERT #F49620-97-1-0459

Parent Award: AFOSR F49620-95-1-0318

AASERT Students: Arrash Hassibi & Meeko Oishi

PI: Stephen Boyd

Two students, Arrash Hassibi and Meeko Oishi, were supported during the period of this grant. After Arrash Hassibi received his doctorate in August 2000, the AASERT award was used to support a starting student, Meeko Oishi, also an American citizen, who worked on control of hybrid systems.

1 Research by Arrash Hassibi

Dr. Hassibi worked on several topics, all related to the topic of the AASERT award and its parent AFOSR grant. All of his research work is either published and available publicly, or will be published soon.

1.1 Advanced Optimization for GPS Applications

In this research, the problem considered was the parameter estimation in linear models when some of the parameters are known to be integers. Such problems arise, for example, in positioning using carrier phase measurements in the global positioning system (GPS), where the unknown integers enter the equations as the number of carrier signal cycles between the receiver and the satellites when the carrier signal is initially phase locked.

Given a linear model, problems were addressed: (1) The problem of *estimating* the parameters, and, (2) the problem of *verifying* the parameter estimates. It was shown that with additive Gaussian measurement noise:

- The maximum likelihood estimates of the parameters are given by solving an integer least-squares problem. Theoretically, this problem is very difficult computationally (NP-hard).
- Verifying the parameter estimates (computing the probability of estimating the integer parameters correctly) requires computing the integral of a Gaussian probability density function over the Voronoi cell of a lattice. This problem is also very difficult computationally.

However, by using a polynomial-time algorithm due to Lenstra, Lenstra, and Lovász (LLL algorithm),

- The integer least-squares problem associated with estimating the parameters can be solved efficiently in practice.
- Sharp upper and lower bounds can be found on the probability of correct integer parameter estimation.

These results are reported in [HHB98, HB96] and have been successfully applied in spacecraft formation flying using GPS sensing as reported in [CRA+97, CRA+98] which was awarded as the best student paper in the ION GPS-97 conference. The proposed methods for integer parameter estimation in linear models are general and can be applied to applications beyond GPS. These include radar imaging, magnetic resonance imaging (MRI), and communications. The paper [MTHBC98] describes the applications of these methods in communication over multi-input/multi-output channels.

1.2 Low-Authority Controller Design via Convex Optimization

This research addressed the problem of low-authority controller (LAC) design. The premise is that the actuators have limited authority, and hence cannot significantly shift the eigenvalues of the system. As a result, the closed-loop eigenvalues can be well approximated *analytically* using perturbation theory. These analytical approximations may suffice to predict the behavior of the closed-loop system in practical cases, and provide at least a very strong rationale for the first step in the design iteration loop.

The main use of LAC is in lightly damped large structures with an infinite number of elastic modes, where LAC is used to provide a small amount of damping in a wide range of modes for maximum robustness. A high-authority controller (HAC) is then used around the LAC to achieve high damping or mode-shape adjustment in a selected number of modes to meet performance requirements.

We have shown that LAC design can be cast as linear and semi-definite programming problems that can be solved efficiently in practice using interior point methods. Also, we have shown that by optimizing the absolute sum norm of the feedback gains, we can arrive at sparse designs, *i.e.*, designs in which only a small number of the control gains are nonzero. Thus, in effect, we can also solve actuator/sensor placement or controller architecture design problems. These results have been published in [HHB99a].

1.3 Lyapunov Methods for the Analysis of Nonlinear, Hybrid, and Asynchronous Dynamical Systems

This research developed a new theory and control methodology for analyzing *piecewise-linear*, *hybrid*, and *asynchronous* systems, which are systems that are far more complex than those of traditional control theory. Control systems theory to date has mainly concentrated on synchronous systems with continuous state, where the state dynamics are governed by ordinary differential (or difference) equations, and the signals are transmitted perfectly–none are lost, and if there is any delay at all, it is fixed.

However, many of today's control systems are logic-based, and therefore incorporate discrete state dynamics as well as continuous state dynamics. Moreover, due to advances in communication systems and data networks, more and more systems are built over asynchronous (packet-switched) networks where signals can be lost or delayed by varying amounts. These systems and many others such as timing circuits, computer disk drives, multi-mode systems, parallelized numerical algorithms, and queuing networks, do not fall within the framework of continuous synchronous control theory, and therefore, a new theory and control methodology is required for analyzing such

practically important systems.

These so-called complex systems can exhibit very complex behavior, so it is not surprising that, as in robust control, many problems in this area are known or conjectured to be theoretically difficult to solve (NP-hard) or even impossible to solve (undecidable). Therefore, it is not expected to solve these problems exactly in practice. It is expected, however, to develop methods that are effective for most instances of these problems. These are methods that guarantee their results when they work, but are not guaranteed to work for all input data. One such method, which has been widely used successfully in robust control, is to search over a fixed, finite dimensional class of Lyapunov functions that guarantee some specification for a given system—it may not be possible to find such a function, but if one is found, the result is unambiguous. This research extends this Lyapunov function based analysis to the more complex dynamical systems described above.

Specifically, we have developed a new Lyapunov-based theory and practice for three very important classes of complex systems for which there are no existing effective analysis methods: 1) *piecewise-linear systems*, which can model many nonlinear systems, 2) *hybrid systems*, which are continuous systems with discrete logic and memory, and, 2) *asynchronous systems*, which are continuous systems driven by discrete events. In each case, we have developed a class of Lyapunov functions for analyzing such systems. Moreover, we have shown that searching over the proposed class of Lyapunov functions to prove some specification (e.g., stability) can be cast as optimization problems involving linear matrix inequalities, which can then be solved efficiently using widely available software. Examples were shown that demonstrate the effectiveness of the approach and significant improvement over the very few other existing methods. These results are reported in [HBH99, HBH99, HBHed] and in the thesis [Has00].

The analysis methods for hybrid systems have been extended to output feedback control synthesis in [HHB99b, RHH00]. In addition to this, new approaches to the problem of controlling systems over networks with communication delays have been developed, using a jump linear systems approach. This has appeared in [XHH00].

2 Research by Meeko Oishi

Meeko Oishi, whose principal advisor is Professor Claire Tomlin, worked on the topic of switching in nonminimum phase systems with applications to a VSTOL aircraft, as relates to hybrid systems.

2.1 Switching in Nonminimum Phase Systems

Hybrid systems involve continuous-time systems which have many discrete modes of operation. The mathematical framework of hybrid systems allows analysis of complex systems which cannot be accurately modeled or analyzed with continuous-time or discrete-event tools alone. The VSTOL aircraft model we have developed is a good platform on which we can test and develop new control techniques, since it is a high-dimensional, nonlinear system with physical constraints. We have developed a new way to combine restrictions found in physical systems with our knowledge of the dynamics of the system, to guarantee "safe" operation while maintaining performance requirements. For the VSTOL aircraft, we would like to continue to track a desired trajectory in the face of actuator saturation and flight restrictions on such parameters as ground speed, pitch angle, and

flight path angle. Additionally, we would like to be able to easily compute these "safe" areas of operation.

In a paper we published in the 2000 American Controls Conference, [OT00], we proved bounded tracking across mode switches for this non-minimum phase system. The longitudinal dynamics of a VSTOL aircraft were modeled as a nonlinear hybrid system with three modes corresponding to typical aircraft behavior. Due to the nonminimum phase property of the aircraft, exact feedback linearization does not stabilize the aircraft. Using dynamic extension, approximate feedback linearization provides a stabilizing control in each mode. We showed that the system will track with bounded error in each mode as well as across mode switches. Two examples of common VSTOL maneuvers demonstrated the effectiveness of the switched control scheme.

3 Accomplishments/ New findings

As explained in the previous section, progress has been made on the following subjects:

- Advanced methods for resolving integer ambiguities in carrier phase GPS
- Real-time accuracy and integrity monitoring (RAIM).
- GPS sensing in spacecraft formation flying.
- Low-authority controller design.
- Analysis and control of piecewise-linear systems.
- Analysis and control of hybrid systems systems.
- Analysis and control of asynchronous systems over networks.

4 Personnel supported

Arash Hassibi and Meeko Oishi were supported as Research Assistants.

5 Publications

On GPS [HHB98, HB96, CRA+97, MTHBC98, HB98a], on low-authority control [HHB99a], on piecewise-linear systems [HB98b], on hybrid systems [HBH99, HBH99, HBHed, XHH00, OT00, RHH00].

6 Honors/Awards

Best student paper award in ION GPS-97 [CRA+97].

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